Skills Review, part 2

Learning Objectives

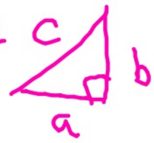
1. Use the Distance Formula
2. Use the Midpoint Formula
3. Find Intercepts From a Graph.
4. Find Intercepts From an Equation.
5. Test an Equation for Symmetry with Respect to the *x*-Axis, the *y*-Axis, and the Origin
6. Calculate the slope of a line.
7. Find the equation of a line in slope-intercept form.

Learning Objectives

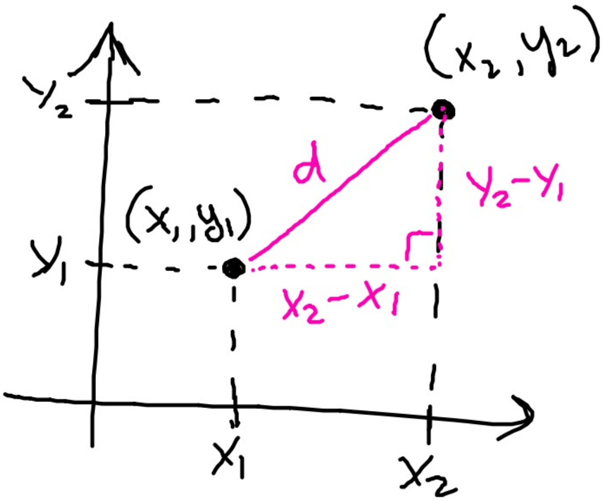
8. Graph horizontal and vertical lines.
9. Solve a problem that can be modeled by a linear equation.
10. Graph circles with the equation given in standard form.
11. Identify the center and radius of a circle from its equation or graph.

Distance Between Two Points

$a^2 + b^2 = c^2$



Use Pythagorean Theorem!



$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$$

Theorem 1

Distance Formula

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \textbf{(1)}$$

Example

Using the Distance Formula

Find the distance *d* between the points (−4, 5) and (3, 2).

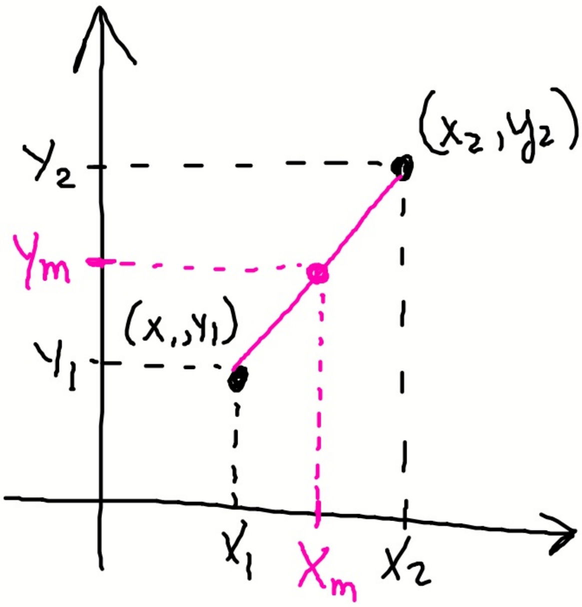
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - (-4))^2 + (2 - 5)^2}$$

$$= \sqrt{7^2 + (-3)^2} = \sqrt{58} \text{ (exact)}$$

$$\approx 7.62 \text{ (approx.)}$$

Midpoint Between Two Points



(average x, average y)

$$x_m = \frac{x_1 + x_2}{2}$$

$$y_m = \frac{y_1 + y_2}{2}$$

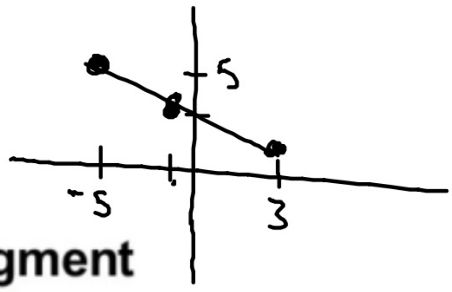
Theorem 2

Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \qquad \textbf{(2)}$$

Example



Finding the Midpoint of a Line Segment

Find the midpoint of the line segment from $P_1 = (-5, 5)$ to $P_2 = (3,1)$. Plot the points P_1 and P_2 and their midpoint.

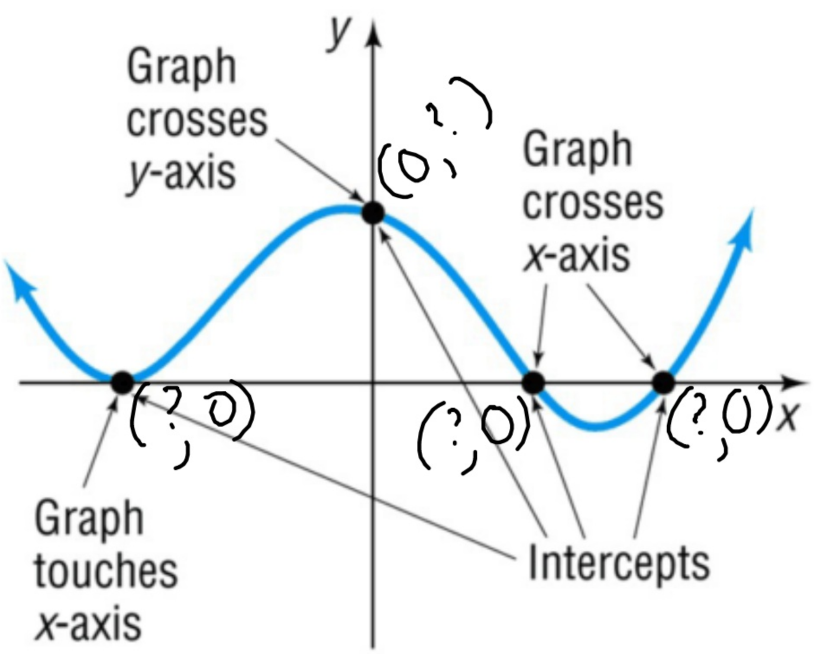
$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-5+3}{2}, \frac{5+1}{2}\right) = (-1, 3)$$

Intercepts

x-int's → $y=0$

y-int's → $x=0$



Example

Finding Intercepts from an Equation

Find the *x*-intercept(s) and *y*-intercept(s) of the graph of $\boxed{y = x^2 - 4.}$ Then graph $y = x^2 - 4$ by plotting points.

X-int's → $y = 0$

$0 = x^2 - 4$

$4 = x^2$

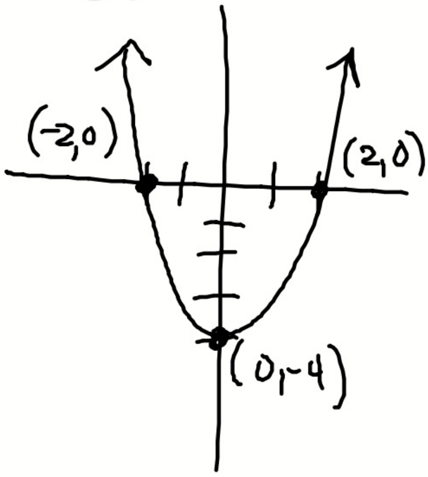
$\pm 2 = x$ $(2,0)$ $(-2,0)$

y-int's → $x = 0$

$y = 0^2 - 4$

$y = -4$

$(0,-4)$



Definitions

X-axis Symmetry (Reflects over X-axis)

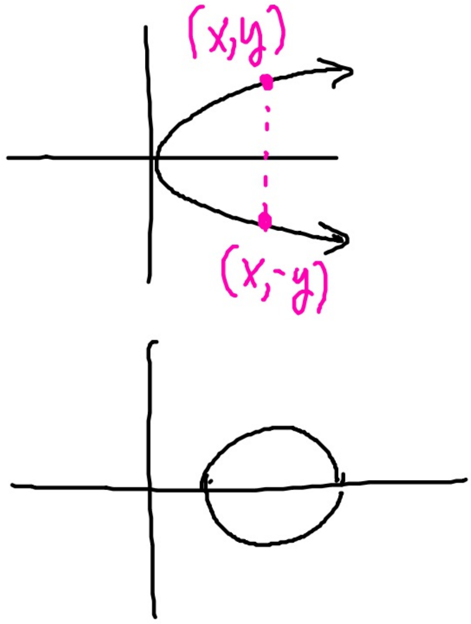
The graph is said to be **symmetric with respect to *x*-axis** if, for every point (x, y) on the graph, the point (x, $-y$) is also on the graph.

The graph is said to be **symmetric with respect to the *y*-axis** if, for every point (x, y) on the graph, the point ($-x$, y) is also on the graph. y-axis symmetry (reflects over y-axis)

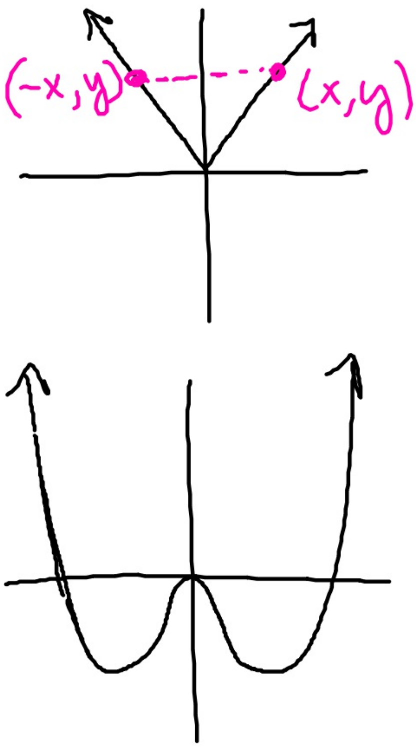
A graph is said to be **symmetric with respect to the origin** if, for every point (x, y) on the graph, the point ($-x$, $-y$) is also on the graph origin symmetry (rotation of 180° around origin)

Examples

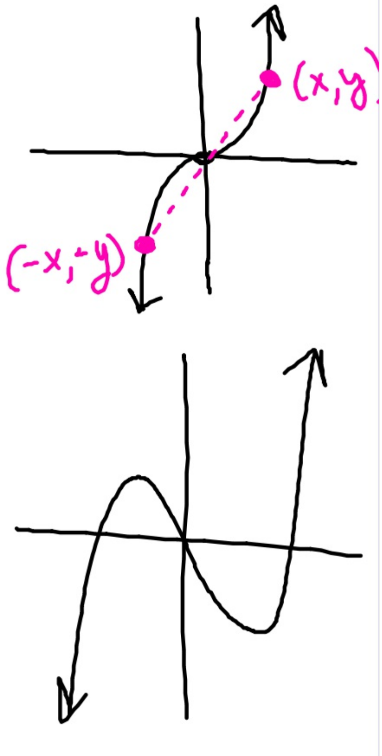
x-axis symm.



y-axis symm.



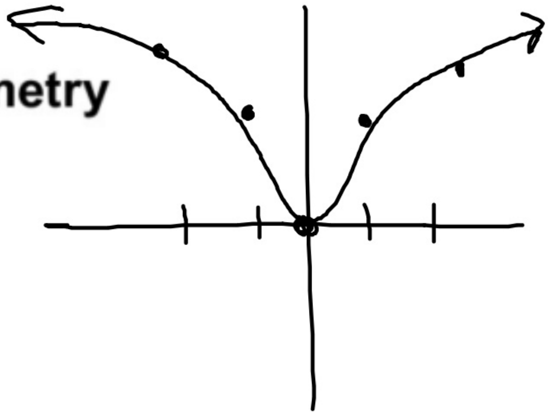
origin symm.



Example

Testing an Equation for Symmetry

Test $y = \dfrac{4x^2}{x^2+1}$ for symmetry.



x	y
-2	16/5
-1	2
0	0
1	2
2	16/5

y-axis symmetry

$(x, y) \to (-x, y)$

"Δ" delta
"change"

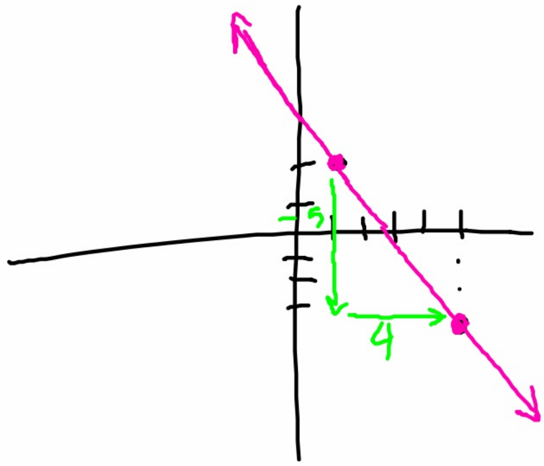
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example

Finding and Interpreting the Slope of a Line Given Two Points

Find the slope *m* of the line containing the points (1, 2) and (5, −3). Graph the line.

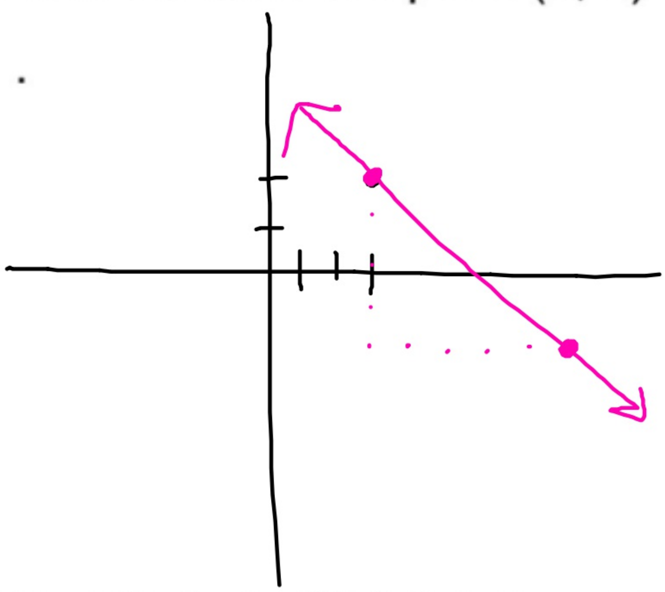
$$m = \frac{-3-2}{5-1} = \frac{-5}{4}$$



Example

Graphing a Line Given a Point and a Slope

Draw a graph of the line that contains the point (3, 2) and has a slope of $-\dfrac{4}{5}$.



Example $y = mx + b$ / $y - y_1 = m(x - x_1)$

Finding an Equation of a Line Given Two Points

Find an equation of the line containing the points (2, 3) and (−4, 5). Graph the line.

First: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - 3}{-4 - 2} = \dfrac{2}{-6} = -\dfrac{1}{3}$

Next: $y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{3}(x - 2)$

$y - 3 = -\frac{1}{3}x + \frac{2}{3}$

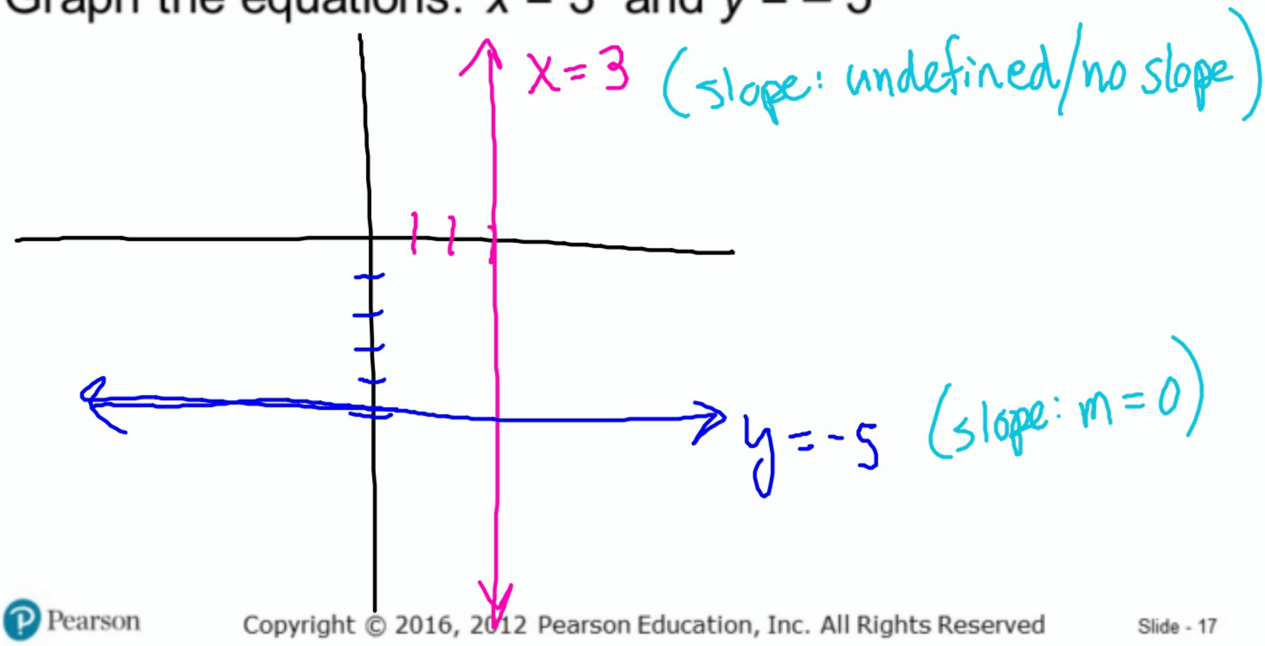
$y = -\frac{1}{3}x + 3\frac{2}{3}$

$\boxed{y = -\frac{1}{3}x + \frac{11}{3}}$

Example

Graphing Lines

Graph the equations: $x = 3$ and $y = -5$



Application

- An Uber driver charges his customers \$5 for a ride plus \$0.75 per mile.
- a) Find a linear equation to model the cost C of riding m miles.

$C = .75m + 5$

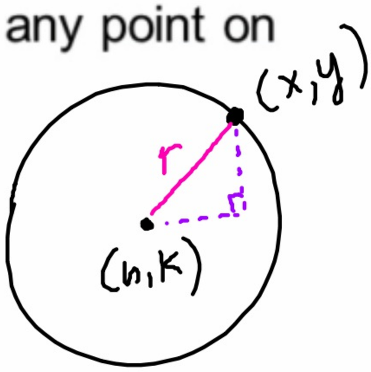
b) Find the cost of a 20 mile ride with this Uber driver.

$C = .75(20) + 5 = \$20$

Definition

A **circle** is a set of points in the *xy*-plane that are a fixed distance *r* from a fixed point (*h*, *k*). The fixed distance *r* is called the **radius**, and the fixed point (*h*, *k*) is called the **center** of the circle.

*Find the distance from the center (*h*,*k*) to any point on the circle (*x*,*y*) using the distance formula.



$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$r=\sqrt{(x-h)^2+(y-k)^2}$$

$$\boxed{r^2=(x-h)^2+(y-k)^2}$$

standard form equation

Example

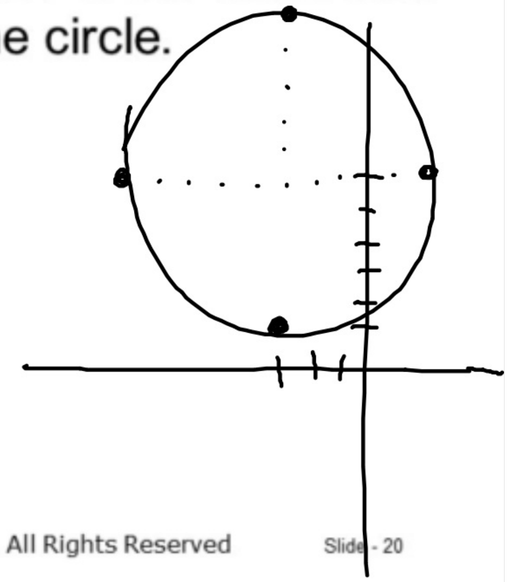
Writing the Standard Form of the Equation of a Circle

Write the standard form of the equation of the circle with radius 5 and center (−3, 6). Graph the circle.

r h k

$$(x-h)^2+(y-k)^2=r^2$$

$$\boxed{(x+3)^2+(y-6)^2=25}$$



Example

Finding the Intercepts of a Circle

For the circle $(x+3)^2+(y-2)^2=16$, find the intercepts, if any of its graph.

y-ints: let $x=0$... $(0+3)^2+(y-2)^2=16$

$9+(y-2)^2=16$

$(y-2)^2=7$

$y-2=\pm\sqrt{7}$

$y=2\pm\sqrt{7}$

$(0, 2\pm\sqrt{7})$

$$(x+3)^2 + (y-2)^2 = 16$$

center: $(-3, 2)$

radius: 4

x-ints: $y=0$

$$(x+3)^2 + (0-2)^2 = 16$$

$$(x+3)^2 = 12$$

$$x+3 = \pm 2\sqrt{3}$$

$$x = -3 \pm 2\sqrt{3}$$

$$\boxed{(-3 \pm 2\sqrt{3}, 0)}$$

General Form of a Circle – expand standard form and set equal to zero

$$\boxed{(x-h)^2+(y-k)^2=r^2}$$ ← standard

$$\boxed{x^2+y^2+Cx+Dy+E=0}$$ ← general form

Ex: $(x+3)^2+(y-2)^2=16$

$$x^2+6x+9+y^2-4y+4=16$$

$$\boxed{x^2+y^2+6x-4y-3=0}$$